Analyzing the reliability of shuffle-exchange networks using reliability block diagrams

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Supercomputers and multi-processor systems are comprised of thousands of processors that need to communicate in an efficient way. One reasonable solution would be the utilization of multistage interconnection networks (MINs), where the challenge is to analyze the reliability of such networks. One of the methods to increase the reliability and fault-tolerance of the MINs is use of various switching stages. Therefore, recently, the reliability of one of the most common MINs namely shuffle-exchange network (SEN) has been evaluated through the investigation on the impact of increasing the number of switching stage. Also, it is concluded that the reliability of SEN with one additional stage (SEN+) is better than SEN or SEN with two additional stages (SEN+2), even so, the reliability of SEN is better compared to SEN with two additional stages (SEN+2). Here we re-evaluate the reliability of these networks where the results of the terminal, broadcast, and network reliability analysis demonstrate that SEN+ and SEN+2 continuously outperform SEN and are very alike in terms of reliability.

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1. Introduction

Demands for even more computing power have never ceased. Although the performance of processors has been doubled in approximately every three-year span from 1980 to 1996, some important issues were revealed, whose solutions would require a huge amount of computing power. In 1987, the U.S. government office of science and technology policy defined many grand challenging problems as fundamental applications to science or engineering, whose solutions would be enabled by applying high-performance computing resources that could become available in the near future. Some of these problems include: Computational fluid dynamics: long-range weather prediction, global climate change, computational ocean sciences, enhanced oil and gas recovery, nuclear reactor design, automobile and hypersonic aircraft design, and quiet submarines. Electronic structure calculations for the design of new materials: chemical catalysts, immunological agents, drug design, human genome, semiconductors, and superconductors. Plasma dynamics for fusion energy applications and military: nuclear fusion, combustion systems, air, sea, and undersea surveillance for safe and efficient military technology. Calculations to understand the fundamentals of matter: quantum chromodynamics, astrophysics, structural analysis, seismology, and condensed matter theory. Symbolic computational: speech recognition, natural language processing, computer vision, image processing, automated reasoning, data mining for modeling business and financial processes, and discrete and continuous simulations of design, manufacturing, and production issues, e.g., in transportation systems. In order to solve these challenging problems, the goal is to obtain computer systems capable of computing at the 1012 floating-point operations per second (teraflops). Even the smallest of these problems requires gigaflops of performance for hours at a time, the larger problems require teraflops performance for more than a thousand hours at a time [1,2]. Single-processor supercomputers have achieved unheard of speeds and have been pushing hardware technology to the physical limits of cheap manufacturing. However, this trend will reach soon to an end, because there are physical and architectural bounds that limit the computational power of a single-processor system. Regarding the processing speed limitation of sequential computers and the easy and cost-effective availability of VLSI technology, employing a large number of processors to accomplish a given computation is an alternative. Parallel computers with multiple processors are opening the door to teraflops computing performance to meet the increasing demand of computational power. The main argument for using multi-processors is to create powerful computers through simply connecting them by multiple
processors. A multi-processor system is expected to reach faster speed than the faster single-processor system. Furthermore, a multi-processor system consisting of a number of single processors is expected to be more cost-effective compared with building a high-performance single processor. Another advantage of a multi-processor system is fault-tolerance, if a processor fails, the remaining processors should be able to provide continued service, albeit with degraded performance [1,3,4].

Given the above discussion, several questions may arise; how do these processing elements communicate and cooperate? In which fashion data should be transmitted between individual processors? What sort of interconnection is provided? In other words, a parallel computer requires some kinds of sub-system communications to interconnect processors, memories, disks, and other peripherals. The task of communicating between different nodes is the responsibility of the interconnection networks. An interconnection network is a system of switches and links that connects N input channels to M output channels that can be used for internal connections among processors, memory modules and I/O devices. Therefore, the design of an efficient interconnection network is very critical for the construction of efficient multi-processor systems [1,5].

With the increasing number of nodes in a supercomputer environment, it seems that the desirable option is to use the multistage interconnection networks (MINs), since MINs are able to provide a good performance at a relatively low cost [1,6,7]. Therefore, MINs are often used in the context of SIMD (single-instruction multiple-data) and MIMD (multiple-instruction multiple-data) parallel machines and are also increasingly adopted for implementing the switching fabric of high-capacity communication processors, including ATM switches, gigabit Ethernet switches, and terabit routers [7,8]. For instance, MINs are frequently used to connect the nodes of IBMSP [9] and CARY X-MP series [10].

Generally, MIN’s structure consists of two parts: sources (inputs) and destinations (outputs). These sources and destinations are connected by multi-stage switching elements so that all sources have access to all destinations [1,11].

MINs can be divided into two categories: single-path MINs and multiple-path MINs. Generally, single-path MINs are built from switching elements of size 2 \( \times \) 2. This minimization helps to reduce the hardware costs in these networks. The numbers of switching stages in single-path MINs of size \( N \times N \) is \( (\log_2 N) \) and in each stage there are \( (N/2) \) switching elements. The network complexity (total number of switching elements in MINs) of single-path MINs is \( (2 \log_2 N) \), which is a reasonable network complexity compared to the network complexity \( (N^2) \) of the crossbar [11–13]. Thanks to the low cost of single-path MINs, many networks such as shuffle-exchange network (SEN) [14], Baseline [15], and Generalized Cube [16] are an excellent choice for large-scale systems. But a major problem in this kind of MINs is that there is only one path between each source-destination pair. Thus, the switches across the path becomes unavailable (faulty or busy), the entire network collapses. The solution would be to increase the fault-tolerance [13,17,18].

The basic idea of MIN’s fault-tolerance is to increase the number of paths between each source-destination pair, so that alternate paths can be taken in case of unavailability. A method to create redundancy in the single-path MINs is to increase the number of switching stages [11,14,18,19].

SEN is a single-path MIN, its double-path version is named SEN+ (with one additional stage), and its quadruple-path is SEN+2 (with two additional stages). The analysis of the three networks SEN, SEN+, and SEN+2 shows that the reliability of SEN+ is more than two other networks and also reliability of SEN is much higher than that of SEN+2 [20]. These results prove quite surprising; because, as mentioned, single-path MINs are the most vulnerable MINs, and fail with a single fault (the minimum fault possible) and it was unexpected for SEN to be more reliable than SEN+2. The main reason for poor results regarding SEN+2 is higher network complexity in comparison with the other two [20]. It should be noted that these results are very influential and can be extended to other types of single-path MINs because these MINs are equivalent in terms of topology [11,19,21].

In this paper, we will analyze the three reliability parameters, terminal, broadcast, and network reliability for SEN, SEN+, and SEN+2. As we shall observe, analytical results indicate that both SEN+2 and SEN+ perform very alike in terms of reliability. But both of them are always more reliable compared to the single-path SEN.

The rest of this paper is organized as follows: motivation, a vision of related works, contribution, and structure of SEN, SEN+, and SEN+2 will be presented in Section 2. Three reliability parameters, terminal, broadcast, and network reliability will be evaluated for SEN, SEN+, and SEN+2 in Sections 3–5 respectively. At last, some conclusions will be made in Section 6.

2. Background

2.1. Motivation

The main approach for improving the fault-tolerance of MINs is to create redundant paths between each source-destination pair. On the other hand, increasing the number of stages is one of the major ideas for creating redundancy in MINs’ paths [14,18,19]. However, reliability should be considered as well. The question that arises is what the impact of increasing the number of stages is on reliability. Previous analyzes have shown that one extra stage has positive impact on the reliability of MINs [14,22,23]. However, here another argument is the impact of increasing one more stage on the achieved reliability. To answer these questions, the reliability of three MINs, SEN, SEN+ (SEN with one extra stage), and SEN+2 (SEN with two extra stages) have been recently analyzed which leads to the following conclusions [20]:

1) SEN+ always has a higher reliability than SEN and SEN+2.
2) SEN achieves much higher reliability than SEN+2.

The first result is quite expected as it may be due to high network complexity of the SEN+2. But, the second result is surprising, since in previous studies, single-path MINs have been considered as the most unreliable networks, and they fail in case of a single fault [11,14,18,19,22–24]. In addition to the unexpected conclusion [20], there are also some other issues

1) More reliability equations are computed only for a small size of network, \( 8 \times 8 \).
2) There are complex relationships between the network components that usually use tools like reliability block diagrams in order to get a correct result [23,24]. However, it is limited only to a simplified description of the relationships that cannot be used to determine the reliability of the MIN [20].

In summary, according to the above discussions, our motivation here is to re-analyze the reliability of three aforementioned networks with more details. The analysis conducted in this research will show that single-path SEN is one of the most unreliable MINs.

2.2. Related works

Generally, a network can be defined as a collection of nodes and links (which are known as vertices and edges in graph theory,
respectively) in which some particular nodes are called terminals [39,49]. On the other hand, the reliability of a network is defined as the connectivity probability of certain set of terminal nodes with each other. This connection can be achieved with at least one fault-free path between the nodes. If this connection is achieved, then the network is in the state of up, otherwise it is in the state of down [13,14,23,39,40–43]. However, the connectivity analysis is very challenging in the case of complex networks. Complex networks are consisting of multiple source and destination nodes, complex topology, interdependencies at the component and system levels, and uncertainties in actual conditions of network components and deterioration models [40,41]. According to this definition, lifeline networks such as electrical and gas networks [40,41,48], wireless mobile ad hoc networks (MANETs) [42], wireless mesh networks [50–53], wireless sensor networks [54,55], nano-sensor based on nano-wired networks [44], social networks [45], stochastic-flow manufacturing networks (SMNs) [46], and interconnection networks [13,20,22,23] are known as complex network systems from the viewpoint of reliability.

According to the reported researches, reliability investigation of the complex networks can be accomplished by simulation or analytical models. Although simulation-based approaches are easily implemented, there are some restrictions to their effectiveness. For instance, the number of performed simulations should be large enough to provide a comprehensive study which can be extremely time-consuming. Furthermore, simulation presents a small range of results compared to the analytical methods. Clearly, analytical methods have been avoided due to their complexity in favor of the simplicity of using simulation. Using reliability equations, analytical methods have been developed to present an exact solution for computing the reliability of a system. Therefore, the time-consuming calculations and the non-repeatability issue of the simulation methodology should be eliminated. Given the reliability equation for a system, further analyses on the system such as computing exact values of the reliability, failure rate at specific points in time, computation of the system MTTF (mean time to failure) can be performed. In addition, reliability optimization techniques can be utilized to promote design improvement efforts. In the context of complex networks, so far attempts have been made to achieve an analytical assessment of the reliability. In what follows, we review some recent important works in this field.

The Recursive Decomposition Algorithm (RDA) has been developed in Ref. [47] that recursively identified disjoint link/cut sets to calculate the reliability of a node pair connection in large networks. This method was applied to the lifeline networks such as an electric power network and a water supply network to estimate seismic reliability. RDA has merits when it is used in the rapid risk assessment of multiple hazard scenarios. However, its application is limited to networks with one initial node and one terminal node. Therefore, in Ref. [41] an analytical method named Logical Expansion of RDA (LE-RDA) has been proposed for reliability analysis of any generic networks, especially for lifeline networks. The LE-RDA generalized the recursive decomposition process to be applicable to complex system definitions of multiple node pairs. It was found that the LE-RDA has some advantages over Monte Carlo Simulations (MCS) based methods in rapid risk assessment of lifeline networks. However, the limitations of the non-simulation based method of LE-RDA include additional implementation costs for large networks with redundant paths considering a full set of link sets. An analytical method called Matrix-based System Reliability (MSR) was proposed in Ref. [43], which calculated the probability of general system events with correlated system components using efficient matrix manipulations and minimal set identification. This method is applicable not only to single node pair network reliability problems but also to any general systems including non-network-graph based systems. However, unlike RDA, the MSR method requires predefined system definitions for network analysis. In addition, the MSR method has limitations in the size of a system as it requires an exponentially increasing matrix size to fully represent all existing minimal disjoint sets in a large system. Ref. [44] focuses on the reliability of one particular nanosensor (the hydrogen gas nanosensor of ultra-small palladium nanowires). The proposed framework in Ref. [44] is applicable to any nanosensor or nanodevices of size $n \times m$ with a square lattice structure operating in a dynamic environment. In this work, it analytically obtains the reliability functions $R_{x \times m}(x)$ and $R_{n \times m}(x)$, from which it derived the expected lifetime and variance of each: $E[X_{x \times m}], E[X_{n \times m}]$, var[$X_{x \times m}$] and var[$X_{n \times m}$]. However, it is hard to compute the exact reliability function $R_{n \times m}(x)$ when $n > m$. It derives bounds for $R_{n \times m}(x)$, $E[X_{x \times m}]$, and var[$X_{n \times m}$] rather than an exact reliability analysis for this case. Ref. [14] examines the so-called all-terminal network reliability of the SEN and SEN+ which addresses the probability that at least one path exists between each source and every destination. In this work, an exact reliability equation for the SEN of size $N \times N$ was obtained, which is one of the simplest MINs in terms of reliability analysis. Also, relatively accurate reliability equations were calculated for the SEN+ of size $8 \times 8$ and $16 \times 16$ using the continuous time Markov chains (CTMC) approach. However, as network size increases, reliability modeling of SEN+ using CTMC is obviously complex. Therefore, an approximation technique for determining the reliability of the larger SEN+ was considered by providing reliability bounds. Although the work has been done in Ref. [14] is relatively appropriate for less complex networks such as the SEN and SEN+, but it cannot be recommended for other complex networks because of the following reasons: The major problem with CTMC approach is the exponential growth of the state space as the network size increases. The state space can be reduced somewhat by some assumptions in the case of less complex networks such as the SEN+. However, due to the high complexity of most MINs, it is very prone to error even for small network sizes. On the other hand, as shown in Ref. [14], the lower bound reliability was relatively close to the exact reliability to network size $16 \times 16$. However, as the network size increases, these reliability bounds cannot obtain accurate results and are very prone to error, even for less complex networks. Ref. [27] evaluates the reliability of extra-stage cube (ESC), which is a single-path MIN with adding an extra stage along with multiplexers and demultiplexers at input and output stages, respectively. In this work, almost exact reliability equations were obtained for terminal and broadcast reliability using the reliability block diagrams. However, similar to Ref. [14], the reliability bounds were used for determining the network reliability of the ESC network, which cannot be an accurate method, especially for large network sizes. In addition, in this study, the reliability of the multiplexers and demultiplexers were not considered in determining the reliability of the entire network. In Ref. [26], the terminal reliability of a certain class of MINs, named gamma network, evaluated. The reliability block diagrams of the gamma network are designed and utilized to obtain a lower and upper terminal reliability bounds. The reliability bounds methodology presented in this work for network size $N \times N$ provides reasonable estimates for terminal reliability of the gamma network of size $8 \times 8$. However, as the network size increases, this methodology is highly prone to produce false information for reliability of the networks. In addition, gamma network is made of switches with different sizes, which must be taken into account in a comprehensive reliability analysis of the network which was not considered in Ref. [26]. Also, terminal reliability is merely assessed, while both broadcast and network reliability is also important for a comprehensive reliability study of such networks. In order to improve the reliability of SEN, a new
MIN architecture called shuffle exchange network with minus one stage (SEN−) has been proposed in Ref. [33]. It is assumed that the reliability can be improved by reducing one stage of the network. However, it is completely a false assumption, since some of the source nodes cannot reach some of destination nodes when one stage of the SEN is reduced. Therefore, network reliability which is connectivity of all the source nodes to all destinations nodes is violated with this assumption at the initial state.

According to the works reported in the field of complex networks and MINs, reliability analysis can be considered from three different perspectives: (1) Reliability analysis method, (2) the parameters considered for reliability analysis (terminal/broadcast/network), and (3) network size. In what follows these three perspectives are clarified.

In the past, many reliability analyses were conducted based on two main approaches: continuous time Markov chains (CTMC) [14,23,25] and reliability block diagram (RBD) method [24,26,27]. The major problem with the CTMC approach is the exponential growth of the state space as the network size increases [14,23,28]. In MINs, we need to consider the operational status of each switching elements in each stage. For instance, the 8 × 8 SEN has 16 switching elements, so we have 2^{16} possible states to be considered. In the past, some of the assumptions used to reduce the number of these states to reduce the complexity of the problem [14,25]. However, this makes us to obtain inaccurate analysis of the network, even for small network sizes. The next approach is to use RBD method. RBDs are intuitive and transparent methods to describe the reliability of safety critical systems, even for large-scale and complex systems [28]. Therefore, the most researchers in this context, believe that the use of RBDs is one of the most essential steps for system reliability analysis [29–31]. However, to the best of our knowledge, so far, a simple series-parallel RBDs has been used to analyze the reliability of MINs [23,24,26,27] that cannot be responsive to the complexity of fault-tolerant MINs. Since the most fault-tolerant MINs should be considered as complex series-parallel RBDs (or complex RBDs), the reliability analysis of the RBDs requires detailed calculations.

In MINs, the communication can be one-to-one, one-to-all, and all-to-all. Therefore, MINs reliability can be assessed in three dimensions: terminal, broadcast, and network reliability. However, often, one of these dimensions has been considered, for example, it is focused only on network reliability [14,23–25] or on terminal reliability [26]. But, we need to consider all three parameters for a comprehensive reliability analysis.

Another important parameter that should be considered in reliability analysis of the complex network systems is the network size. The reliability equations for small-size networks can be useful in analyzing the performance of the networks. However, we need to obtain the reliability equations for larger network sizes in order to get closer to reality and large-scale systems. But the problem here is the increased number of components and the complexity of relations among them in terms of reliability. In fact, one of the main reasons for the use of simulation methods to assess the reliability of complex systems is difficult in analyzing the large-scale systems [41].

2.3. Contribution

According to the above arguments, we basically need accurate information about the components of the system as well as the relationships among those. Clearly, reliability of a system cannot be precisely measured, regardless of the functionality of individual system components and their impact on the system's performance. On the other hand, reliability block diagram (RBD) is the graphical representation of the components of a system and the relationships between them, which can be used to determine the overall system reliability, even for large-scale and complex systems [23,32,37]. Therefore, it can be concluded that the RBD method is an accurate method for the analysis of such systems. Nevertheless, in the area of MINs, we often need to use the complex series–parallel RBDs for a more detailed analysis. In addition, to achieve a comprehensive analysis, we need to examine all of the reliability parameters in the network (i.e. terminal, broadcast, and network reliability). Therefore, in this paper, RBDs will be widely used in details. Also, in order to accurately determine reliability, we will consider a structure as close as possible to reality from the reliability point of view. Then, the complex series–parallel RBDs are used to determine the reliability of the fault-tolerant MINs. In other words, unlike previous works, we do not believe that all relationships between MIN's components should be organized as a simple form. Also, in this paper, we will analyze all aforementioned dimensions of the reliability to get a comprehensive analysis. Therefore, we will be able to provide more accurate results of reliability of the networks using this methodology. Furthermore, as discussed, another important parameter in the reliability analysis is the network size. Therefore, in this paper, all reliability equations for all three measures of reliability namely terminal, broadcast, and network will be calculated for different size N × N.

In summary, our contributions to the body of knowledge are as follows: First, in this paper, we will analyze the reliability of three known MINs SEN, SEN+, and SEN++ in order to evaluate the efficiency of the common method of increasing the number of switching stages on the reliability of the networks. Therefore, the results of this research are very important and can be expanded to other types of single-path MINs because these MINs are equivalent in terms of topology [11,19,21]. Second, the methodology used in this paper to analyze the reliability is superior to the methods used in the past in terms of the following perspectives: (1) In this paper, in contrast to reported works, the RBD is exactly used in conjunction with a novel approach. In Refs. [14,23–27], the RBD is often used for providing reliability bounds, which is an approximation technique for determining the reliability and is prone to error for large-scale networks as well as complex networks. On the other hand, in Refs. [14,23–27] to avoid complex calculations the networks are usually modeled in simple series–parallel RBDs, which can also lead to inaccurate results. In this paper, to deal with these shortcomings, it is not assumed all relationships between MIN's components are organized as a simple form. Therefore, in order to accurately determine reliability, we will consider a structure as close as possible to reality from the reliability point of view. That is, complex series–parallel RBDs are used to determine the reliability of the fault-tolerant MINs. (2) This paper comprehensively analyzes all aforementioned dimensions of the reliability (terminal, broadcast and network reliability). Also, all reliability equations for all three measures of reliability will be calculated for different size N × N. (3) The methodology used in this paper is applicable for analyzing the reliability of other MINs, even with more complex structures.

2.4. Structure of SEN, SEN+, and SEN++

In this sub-section, we get more familiar with the structure of the shuffle-exchange networks.

SENs of size N × N are comprised of (log_2 N) stages, and each stage contains (N/2) switching elements of size 2 × 2. A SEN of size 8 × 8 is shown in Fig. 1. The network complexity of an N × N SEN is (\frac{1}{2} (log_2 N)). In this network, there is only one path between each source–destination pair.

A SEN+ (SEN with one additional stage) of size N × N is comprised of (log_2 N+1) stages, and each stage contains (N/2) switching elements of size 2 × 2. A SEN+ of size 8 × 8 is
shown in Fig. 2. The network complexity of an \( N \times N \) SEN+ network is \( \frac{N}{2}((\log_2 N) + 1) \). In this network, there are two paths between each source–destination pair.

A SEN+2 (SEN with two additional stages) of size \( N \times N \) is comprised of \((\log_2 N) + 2\) stages, and each stage contains \( N/2 \) switching elements of size \( 2 \times 2 \). A SEN+2 of size \( 8 \times 8 \) is shown in Fig. 3. The network complexity of an \( N \times N \) SEN+2 network is \( (N/2)((\log_2 N) + 2) \). In this network, there are four paths between each source–destination pair.

### 3. Terminal reliability of SEN, SEN+, and SEN+2

Terminal reliability shows the reliability between pairs of sources and destinations, defined as the probability of the occurrence of at least one fault-free path between a source–destination pair.

SEN is a single-path MIN, so all switches between a source–destination pair is vital. Terminal reliability RBD of SEN of size \( N \times N \) is shown in Fig. 4.

Let \( r \) be the probability of a switch being operational. Terminal reliability of \( N \times N \) SEN is obtained by the Eq. (1).

\[
R_t(\text{SEN}) = r^{\log_2 N}
\]  

(SEN+) is a double-path MIN. With respect to Fig. 2, terminal reliability RBD of SEN+ of size \( N \times N \) is shown in Fig. 5. Note that RBDs should reflect the characteristics of each network.

RBD in Fig. 5 is composed of three blocks A, B, and C. Reliability for each of these blocks is obtained by the Eqs. (2) and (3).

\[
R(A) = R(B) = r
\]

\[
R(C) = 1 - (1 - r^{\log_2 N - 1})^2
\]

The terminal reliability of the \( N \times N \) SEN+ is calculated as follows:

\[
R_t(\text{SEN+}) = r^2[1 - (1 - r^{\log_2 N - 1})^2]
\]

(SEN+) is a quadruple-path MIN. With respect to Fig. 3, terminal reliability RBD of SEN+2 of size \( N \times N \) is shown in Fig. 6.

First, we will calculate the reliability of parts A and B of Fig. 6, then we can calculate the reliability of the entire network.

\[
R(A) = r^2(1 - (1 - r^{\log_2 N - 2})^2)
\]

\[
R(B) = 1 - (1 - (r^2(1 - (1 - r^{\log_2 N - 2}))^2))^2
\]

The terminal reliability of the \( N \times N \) SEN+2 is calculated by the Eq. (7).

\[
R_t(\text{SEN+2}) = r^2[1 - (1 - (r^2(1 - (1 - r^{\log_2 N - 2}))^2))^2]
\]

From Eqs. (1), (4), and (7), the terminal reliability of the three networks of the size \( 8 \times 8 \) is obtained accordingly.

\[
R_t(\text{SEN}) = r^3
\]
\[ R_t(\text{SEN}+) = r^2[1 - (1 - r^2)^2] \]  
(9)

\[ R_t(\text{SEN}+2) = r^2[1 - (1 - (r(1 - (1 - r^2)))^2)] \]  
(10)

The terminal reliability results for the three networks of size 8 \times 8 are given in Table 1. As we can see SEN has the worst results and SEN+ has the best. These results indicate that SEN+2 operates slightly weaker than SEN+, however, it is much better than SEN.

Results achieved by previous studies [20] for SEN and SEN+ are the same as the results obtained by us. But in the previous investigations [20], the results are unexpectedly inaccurate for SEN+2. The main reason for this could be the conducted reliability analysis method. As mentioned in Section 2, determining the exact relationships among the components of fault-tolerant MINS is intractable [23]. In addition, the result was limited only to a simplified description of the relationships that is not desirable to determine the reliability of the MIN in general [20]. That is, the simplified model may be good to analyze the reliability of simpler systems, such as SEN and SEN+, which have less complexity as compared with the structure of the SEN+2. However, as the complexity of the network increases, the simple approach leads to erroneous results. That is why we need a more proper approach such as the RBD method, which decreases the probability of error to a minimum.

Terminal reliability results shown in Fig. 7 reflect the fact that SEN+ and SEN+2 are very alike and are both more reliable than SEN. We can conclude that adding one stage is more efficient than adding two stages to SEN in terms of terminal reliability. The reason is that terminal reliability of SEN is increased by adding one extra stage, while it almost does not change with the addition of another extra stage.

4. Broadcast reliability of SEN, SEN+, and SEN+2

Broadcast reliability represents the reliability between a given source and all destinations, defined as the probability of successful communication between the source and all network destinations.

<table>
<thead>
<tr>
<th>Switching reliability</th>
<th>( R_t(\text{SEN}) )</th>
<th>( R_t(\text{SEN}+) )</th>
<th>( R_t(\text{SEN}+2) )</th>
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<tr>
<td>0.99</td>
<td>0.970299</td>
<td>0.979712</td>
<td>0.979708</td>
</tr>
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<td>0.98</td>
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</tbody>
</table>

\[ R_{bd}(\text{SEN}) = r^K \]  
(12)

With respect to Fig. 3, broadcast reliability RBD of \( N \times N \text{SEN}+ \) is shown in Fig. 9.

\[ R_{bd}(\text{SEN}+2) = r^{N+2}/[1 - (1 - r^{N+2}/4)^2] \]  
(16)

With respect to Fig. 4, broadcast reliability RBD of \( N \times N \text{SEN}+2 \) is shown in Fig. 10.

Therefore, according to the network structure as discussed earlier, all switches in the last stage are vital in this type of reliability. With respect to Fig. 1, broadcast reliability RBD of \( N \times N \text{SEN} \) is shown in Fig. 8.

\[ K = \frac{\log_{10}N}{\log_{10}2} \]  
(11)

\[ R_{bd}(\text{SEN}) = r^K \]  
(12)

\[ R_{bd}(\text{SEN}+) = r^{N+2}/[1 - (1 - r^{N+2}/4)^2] \]  
(16)

Therefore, according to the network structure as discussed earlier, all switches in the last stage are vital in this type of reliability. With respect to Fig. 1, broadcast reliability RBD of \( N \times N \text{SEN} \) is shown in Fig. 8.

In Fig. 8, \( K \) is calculated using Eq. (11).

\[ K = \frac{\log_{10}N}{\log_{10}2} \]  
(11)

\[ R_{bd}(\text{SEN}) = r^K \]  
(12)

With respect to Fig. 2, broadcast reliability RBD of \( N \times N \text{SEN}+ \) is shown in Fig. 9.

\[ R_{bd}(\text{SEN}+2) = r^{N+2}/[1 - (1 - r^{N+2}/4)^2] \]  
(16)

\[ R_{bd}(\text{SEN}+2) = r^{N+2}/[1 - (1 - r^{N+2}/4)^2] \]  
(16)

With respect to Fig. 3, broadcast reliability RBD of \( N \times N \text{SEN}+2 \) is shown in Fig. 10.

In fact, RBD in Fig. 9 is composed of three blocks A, B, and C. Reliability for each of these blocks is computed as follows:

\[ R(A) = r \]  
(13)

\[ R(B) = 1 - (1 - r^{N-2}/2)^2 \]  
(14)

\[ R(C) = r^{N/2} \]  
(15)

The broadcast reliability of the \( N \times N \text{SEN}+ \) is calculated by the Eq. (16).

\[ R_{bd}(\text{SEN}+) = r^{N+2}/[1 - (1 - r^{N+2}/4)^2] \]  
(16)

With respect to Fig. 3, broadcast reliability RBD of \( N \times N \text{SEN}+2 \) is shown in Fig. 10.

In fact, RBD in Fig. 10 is composed of four blocks A, B, C, and D. Reliability for each of these blocks is given as follows:

\[ R(A) = r \]  
(17)

\[ R(B) = 1 - (1 - r^{N-2}/4)^2 \]  
(18)

\[ R(C) = (1 - r)^2 r^{N-4}/4 \]  
(19)

\[ R(D) = r^{N/2} \]  
(20)

The broadcast reliability of the \( N \times N \text{SEN}+2 \) is calculated as follows:

\[ R_{bd}(\text{SEN}+2) = r^{N+2}/[1 - (1 - r^{N+2}/4)^2] \]  
(16)

Form Eqs. (12), (16), and (21), the broadcast reliability of the three networks for the size 8 \times 8 are obtained as follows:

\[ R_{bd}(\text{SEN}) = r^K \]  
(22)

\[ R_{bd}(\text{SEN}+) = r^{N+2}/[1 - (1 - r^{N+2}/4)^2] \]  
(23)

\[ R_{bd}(\text{SEN}+2) = r^{N+2}/[1 - (1 - r^{N+2}/4)^2] \]  
(24)
The broadcast reliability results for the three networks of size $8 \times 8$ are given in Table 2. Obviously, SEN has the weakest results and SEN+2 has the best. The broadcast reliability results are shown in Fig. 11 for a convenient comparison of networks.

As we can see from Fig. 11, SEN+2 achieves higher broadcast reliability than the other two, especially when the switch reliability is low (0.9–0.95). However, both SEN+2 and SEN+ networks almost show similar results; but they are always at a higher level of broadcast reliability compared to single-path SEN.

Overall, we conclude from this points that although SEN+2 achieves a higher broadcast reliability than the other two networks, adding a stage to SEN is more efficient than adding two stages; because reliability can be increased considerably by adding a stage to SEN, while it is increased only slightly by adding two stages.

5. Network reliability of SEN, SEN+, and SEN+2

Network reliability represents the reliability of the connections between all sources and all destinations, defined as the probability of successful communication between all sources and all network destinations. Therefore, according to the network structure as discussed, all switches in the first and last stages are critical in this type of reliability. With respect to Fig. 2, network reliability RBD of $N \times N$ SEN is shown in Fig. 12.

Network reliability of $N \times N$ SEN is calculated by the Eq. (25).

$$R_0(\text{SEN}) = r^{N/2 \log_2 N}$$  \hspace{1cm} (25)

With respect to Fig. 2, network reliability RBD of $N \times N$ SEN+ is shown in Fig. 13.

The RBD in Fig. 13 is composed of four main blocks 1, 2, 3, and 4. Reliability for each of these blocks can be obtained as follows:

$$R(1) = R(4) = r^{N/2} \tag{26}$$

$$R(2) = (1 - (1 - r)^2)^{N/4} \tag{27}$$

$$R(3) = 1 - (1 - r^{N/4(\log_2 N - 2)})^2 \tag{28}$$

The network reliability of the $N \times N$ SEN+ is calculated by the Eq. (29).

$$R_0(\text{SEN+}) = r^{N(1 - (1 - r)^2)^{N/4}} \left[1 - (1 - r^{N/4(\log_2 N - 2)})^2\right] \tag{29}$$

With respect to Fig. 3, network reliability RBD of $N \times N$ SEN+2 is shown in Fig. 14. Actually, the RBD in Fig. 14 is composed of five main blocks 1, 2, 3, 4, and 5. Reliability for each of these blocks can be obtained as follows:

$$R(1) = R(5) = r^{N/2} \tag{30}$$

$$R(2) = (1 - (1 - r)^2)^{N/4} \tag{31}$$

$$R(3) = 1 - (1 - r^{N/4(\log_2 N - 2)})^4 \tag{32}$$

$$R(4) = 1 - (1 - r^{N/4})^2 \tag{33}$$
The network reliability of the $N \times N$ SEN+2 is calculated by the Eq. (34).

$$R_n(SEN+2)=r^3\left[1-(1-r)^2\right]^2\left\{1-(1-r^4)^2\right\}$$

(34)

From Eqs. (25), (29), and (34), the network reliability of the three networks for the size $8 \times 8$ are obtained as follows:

$$R_n(SEN)=r^{12}$$

(35)

$$R_n(SEN+)=r^8\left[1-(1-r)^2\right]^2\left\{1-(1-r^2)^2\right\}$$

(36)

$$R_n(SEN+2)=r^6\left[1-(1-r)^2\right]^3\left\{1-(1-r^2)^2\right\}$$

(37)

Network reliability results are shown in Table 3. Also comparison of network reliability for the three networks, SEN, SEN+, and SEN+2 of size $8 \times 8$ is shown in Fig. 15.

As it is shown in Fig. 15, SEN+ and SEN+2 always achieve a higher level of network reliability as compared with SEN. In addition, the obtained results of SEN+ and SEN+2 show a very close performance in terms of network reliability. Also, as it is shown in Table 3, it is obvious that SEN+ can be slightly better than SEN+2. Altogether, it can be concluded that adding one switching stage to improve the network reliability of the SEN is much more efficient than adding two switching stages.

While in Ref. [20], it was concluded that the reliability of SEN+ and SEN are much better than the SEN+2, obtained results for terminal, broadcast, and network reliability analysis, demonstrate that both SEN+ and SEN+2 yield similar reliability and are much better than SEN. With respect to the accuracy of reliability analysis method in this paper and the works done in the past, it is believed that results obtained here are closer to reality. These results also reflect the fact that adding one switching stage to SEN is more efficient than adding two switching stages in terms of reliability.

<table>
<thead>
<tr>
<th>Switching reliability</th>
<th>$R_n(SEN)$</th>
<th>$R_n(SEN+)$</th>
<th>$R_n(SEN+2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.99</td>
<td>0.886384</td>
<td>0.922194</td>
<td>0.922194</td>
</tr>
<tr>
<td>0.98</td>
<td>0.784716</td>
<td>0.848749</td>
<td>0.848749</td>
</tr>
<tr>
<td>0.97</td>
<td>0.693842</td>
<td>0.779600</td>
<td>0.779599</td>
</tr>
<tr>
<td>0.96</td>
<td>0.612709</td>
<td>0.714663</td>
<td>0.714661</td>
</tr>
<tr>
<td>0.95</td>
<td>0.540360</td>
<td>0.653832</td>
<td>0.653828</td>
</tr>
<tr>
<td>0.94</td>
<td>0.475920</td>
<td>0.596988</td>
<td>0.596980</td>
</tr>
<tr>
<td>0.93</td>
<td>0.418596</td>
<td>0.543997</td>
<td>0.543984</td>
</tr>
<tr>
<td>0.92</td>
<td>0.367666</td>
<td>0.494716</td>
<td>0.494696</td>
</tr>
<tr>
<td>0.91</td>
<td>0.322475</td>
<td>0.448993</td>
<td>0.448963</td>
</tr>
<tr>
<td>0.90</td>
<td>0.282429</td>
<td>0.406670</td>
<td>0.406629</td>
</tr>
</tbody>
</table>

That is, it is obvious that a significant reliability improvement is provided by adding one stage to SEN, while this is does not sound true by adding another extra stage.
6. Conclusions and future works

In this paper, we analyzed the reliability of three networks, SEN, SEN+, and SEN+ 2 from three perspectives, terminal, broadcast, and network in order to investigate the impact of increasing the number of switching stages on the reliability of MINs. All the analysis on terminals, broadcast, and network reliability achieve almost the same results. These results demonstrate that SEN+ and SEN+ 2 perform the same in terms of reliability. Also, the reliability of these networks is always more than the single-path SENs. Thus, adding one switching stage to SEN is preferable to adding two switching stages in terms of reliability.

The results obtained in Ref. [20] for the SEN+ 2 is different from the obtained results in this paper. That is, in Ref. [20], it was concluded that networks SEN and SEN+ lead to very high reliability compared to the SEN+ 2. Since in all the previous works it has been shown that SEN is one of the most unreliable networks [11,14,18,19,22–25,33] (because it is a non-fault-tolerant MIN) this result is unexpected. Clearly, from the results, SEN gets to lower performance than the other two in terms of reliability.

The results obtained in this paper are further validated from two perspectives: First, it is reasonable and justifiable to the analyzes conducted in Refs. [11,14,18,19,22–25,33]. Second, reliability analysis method undertaken in this paper is much more accurate than the analysis method done in the previous works. Although the simple analysis approach reported in the previous works may useful to study the less-complex networks such as SEN and SEN+, using it in the networks with more complexity such as SEN+ 2 is very prone to misdiagnosis. To cope with this, we have used the RBD method which is often recommended for the precise determination of the reliability of complex systems [29–31].

In total, according to the obtained results, it can be concluded that adding the number of switching stages can lead to more reliability in MINs. But this improvement is limited and may not be responsive to large-scale systems. Therefore, in the future, we need to look for more advanced solutions for improving reliability and fault-tolerance of MINs. Some of the possible solutions could be mentioned in summary as follows [13]; using replicated MINs, using symmetrical MINs, using several MINs in parallel, increasing the size of the switches, and finally using a combination of all of these approaches. On the other hand, we would be faced with several major problems once the new techniques are applied; the first problem is hardware cost [38]. Indeed, one of the reasons for the popularity of method in which switching stages are added to MINs is that this approach increases costs exactly as a single switching stage does, therefore it slightly raises the cost and thus, is cost effective. The next important issue of applying any new technique is to select the appropriate method for reliability analysis. Usually, more advanced techniques lead to more complex topologies. Therefore, in order to achieve a more accurate analysis it is often needed to implement these topologies in complex RDBs (non-series–parallel RDBs). Several methods exist for investigating the reliability of a complex system including [32,34–36]; the decomposition method, the event space method, and the path-tracing method. Another issue is the switching elements heterogeneity. That is, all different switching elements in MINs may not be of the same size and therefore, each one has different reliability that should be considered in the future analysis which makes them further complicated.

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Appendix A. Supplementary materials

Supplementary data associated with this article can be found in the online version at http://dx.doi.org/10.1016/j.ress.2014.07.012.

References


